EXERCISE 1.1

Q.1 Which of the following sets have closure property w.r.t. addition and multiplication

(i) {0}

The set is closed w.r.t. addition because $0 + 0 = 0 \in \{0\}$

The set is closed w.r.t. multiplication because $0.0 = 0 \in \{0\}$

(ii) {1}

The set is not closed w.r.t. addition because $1 + 1 = 2 \notin \{1\}$

The set is closed w.r.t. multiplication because $1.1 = 1 \in \{1\}$

(iii) {0, – 1}

+	0	– 1
0	0	– 1
– 1	– 1	– 2

The set is not closed w.r.t. addition because $-2 \notin \{0, -1\}$

•	0	– 1
0	0	0
– 1	0	1

The set is not closed w.r.t. multiplication because $1 \notin \{0, -1\}$

(iv) {1, – 1}

+	1	– 1
1	2	0
– 1	0	– 2

The set is not closed w.r.t. addition because $-2, 0, 2 \notin \{-1, 1\}$

•	1	– 1
1	1	– 1
– 1	– 1	1

The set is closed w.r.t. multiplication.

Q.2 Name the properties used in the following equations (letters, where used, represents real numbers)

Solution:

(i) $4 + 9 = 9 + 4$ Commutative property w.r.t. '+'

$=$ Commutative property w.r.t. '+'

(ii) $(a + 1) + \frac{3}{4} = a + \left(1 + \frac{3}{4}\right)$ Associative property w.r.t. '+'

(iii) $(\sqrt{3} + \sqrt{5}) + \sqrt{7} = \sqrt{3} + (\sqrt{5} + \sqrt{7})$ Associative property w.r.t. '+'

(iv) $100 + 0 = 100$ Additive Identity

(v) $100 \times 1 = 100$ Multiplicative Identity

(vi) $4.1 + (-4.1) = 0$ Additive Inverse

(vii) $a - a = 0$ Additive Inverse.

(viii) $\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$ Commutative property w.r.t. '.'

(ix) $a(b - c) = ab - ac$ Left distributive property.

(x) $(x - y)z = xz - yz$ Right distributive property.

(xi) $4 \times (5 \times 8) = (4 \times 5) \times 8$ Associative property w.r.t. '.'

(xii) $a(b + c - d) = ab + ac - ad$ Left distributive property

Q.3 Name the properties used in the following inequalities.

Solution:

(i) $-3 < -2 \Rightarrow 0 < 1$ Additive property.

(ii) $-5 < -4 \Rightarrow 20 > 16$ Multiplication property.

(iii) $1 > -1 \Rightarrow -3 > -5$ Additive property.

(iv) $a < 0 \Rightarrow -a > 0$ Multiplicative property.

(v) $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$ Multiplicative property.

(vi) $a > b \Rightarrow -a < -b$ Multiplicative property.

Q.4 Prove the following Rules of Addition

(i) $\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$

Solution:

L.H.S

$= \frac{a}{c} + \frac{b}{c}$

$= a \cdot \frac{1}{c} + b \cdot \frac{1}{c}$ ¤ $\frac{a}{b} = a \cdot \frac{1}{b}$

$= (a + b) \cdot \frac{1}{c}$ Distributive property

$= \frac{a + b}{c}$ ¤ $a \cdot \frac{1}{c} = \frac{a}{c}$

(ii) $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

Solution:

$$\text{L.H.S.} = \frac{a}{b} + \frac{c}{d}$$

$$= \frac{a}{b} \cdot 1 + \frac{c}{d} \cdot 1 \qquad \text{Multiplicative Identity}$$

$$= \frac{a}{b} \cdot \left(d \cdot \frac{1}{d} \right) + \frac{c}{d} \left(b \cdot \frac{1}{b} \right) \qquad \text{Multiplicative Inverse}$$

$$= \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b} \qquad \because \; d \cdot \frac{1}{d} = \frac{d}{d} \;,\; b \cdot \frac{1}{b} = \frac{b}{b}$$

$$= \frac{ad}{bd} + \frac{cb}{db}$$

$$= \frac{ad}{bd} + \frac{bc}{bd} \qquad \text{Commutative Property w.r.t. '.'}$$

$$= ad \cdot \frac{1}{bd} + bc \cdot \frac{1}{bd} \qquad \because \; \frac{a}{b} = a \cdot \frac{1}{b}$$

$$= (ad + bc) \cdot \frac{1}{bd} \qquad \text{Distributive Property}$$

$$= \frac{ad + bc}{bd} \qquad \because \; a \cdot \frac{1}{b} = \frac{a}{b}$$

$$= \text{R.H.S.}$$

Q.5 Prove that $-\dfrac{7}{12} - \dfrac{5}{18} = \dfrac{-21 - 10}{36}$

Solution:

$$\text{L.H.S.} = -\frac{7}{12} - \frac{5}{18}$$

$$= -\frac{7}{12} \cdot 1 - \frac{5}{18} \cdot 1 \qquad \text{Multiplicative Identity}$$

$$= -\frac{7}{12} \left(3 \cdot \frac{1}{3} \right) - \frac{5}{18} \cdot \left(2 \cdot \frac{1}{2} \right) \qquad \text{Multiplicative Inverse}$$

$$= -\frac{7}{12} \cdot \frac{3}{3} - \frac{5}{18} \cdot \frac{2}{2} \qquad \because \; a \cdot \frac{1}{b} = \frac{a}{b}$$

$$= -\frac{21}{36} - \frac{10}{36}$$

$= -21 \cdot \frac{1}{36} - 10 \cdot \frac{1}{36}$ $\because \frac{a}{b} = a \cdot \frac{1}{b}$

$= (-21 - 10) \cdot \frac{1}{36}$ Distributive Property

$= \frac{-21 - 10}{36}$

$=$ R.H.S.

Q.6 Simplify by justifying each step:

(i) $\frac{\mathbf{4 + 16x}}{\mathbf{4}}$

Solution:

$\frac{4 + 16x}{4}$

$= \frac{1}{4} \cdot (4 + 16x)$ $\because \frac{a}{b} = \frac{1}{b} \cdot a$

$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16\,x$ Distributive Property

$= \frac{1}{4} \cdot 4 \cdot 4x$ Multiplicative Inverse

$= 1 + 1 \cdot 4x$ Multiplicative Inverse

$= 1 + 4x$ Multiplicative Identity

(ii) $\frac{\mathbf{\frac{1}{4} + \frac{1}{5}}}{\mathbf{\frac{1}{4} - \frac{1}{5}}}$

Solution:

$\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}$

$= \left(\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}} \right) \cdot 1$ Multiplicative Identity

$$= \left(\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}\right) . 20 . \frac{1}{20}$$ Multiplicative Inverse

$$= \frac{\left(\frac{1}{4}+\frac{1}{5}\right) . 20}{\left(\frac{1}{4}-\frac{1}{5}\right) . 20} \quad \because \quad 20 . \frac{1}{20} = \frac{20}{20}$$

$$= \frac{\frac{1}{4} . 20 + \frac{1}{5} . 20}{\frac{1}{4} . 20 + \frac{1}{5} . 20}$$ Distributive Property

$$= \frac{\left(\frac{1}{4} . 4\right)5 + \left(\frac{1}{5} . 5\right)4}{\frac{1}{4} . 4.5 + \frac{1}{5} . 5.4} = \frac{1.5 + 1.4}{1.5 - 1.4}$$ Multiplicative Inverse

$$= \frac{5+4}{5-4}$$ Multiplicative Identity

$$= \frac{9}{1} = 9$$

(iii) $\dfrac{\mathbf{\frac{a}{b}+\frac{c}{d}}}{\mathbf{\frac{a}{b}-\frac{c}{d}}}$

$$= \left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right) . 1$$ Multiplicative Identity

$$= \left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right) . bd . \frac{1}{bd}$$ Multiplicative Inverse

$$= \frac{\left(\frac{a}{b}+\frac{c}{d}\right) . bd}{\left(\frac{a}{b}-\frac{c}{d}\right) . bd} \quad \because \quad bd . \frac{1}{bd} = \frac{bd}{bd}$$

$$= \frac{\frac{a}{b} . bd + \frac{c}{d} . bd}{\frac{a}{b} . bd - \frac{c}{d} . bd}$$ Distributive Property

$$= \frac{a \cdot \frac{1}{b} \cdot b \cdot d + c \cdot \frac{1}{d} \cdot b \cdot d}{a\frac{1}{b} \cdot b \cdot d - c\frac{1}{d} \cdot b \cdot d} \qquad \because \quad \frac{1}{b} = a \cdot \frac{1}{b} \;,\; \frac{c}{d} = c \cdot \frac{1}{d}$$

$$= \frac{a\left(\frac{1}{b} \cdot b\right)d + c \cdot \left(\frac{1}{d} \cdot d\right)b}{a\left(\frac{1}{b} \cdot b\right)d - c\left(\frac{1}{d} \cdot d\right)b}$$ Commutative Property

$$= \frac{a \cdot 1 \cdot d + c \cdot 1 \cdot b}{a \cdot 1 \cdot d - c \cdot 1 \cdot b}$$ Multiplicative Inverse

$$= \frac{ad + cb}{ad - cb}$$ Multiplicative Identity

(iv) $$\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}\right) \cdot 1$$ Multiplicative Identity

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}\right) \cdot ab \cdot \frac{1}{ab}$$ Multiplicative Inverse

$$= \frac{\left(\frac{1}{a} - \frac{1}{b}\right) \cdot ab}{\left(1 - \frac{1}{a} \cdot \frac{1}{b}\right) \cdot ab} \qquad \because \quad ab \cdot \frac{1}{ab} = \frac{ab}{ab} \cdot$$

$$= \frac{\frac{1}{a} \cdot ab - \frac{1}{b} \cdot ab}{ab - \frac{1}{a} \cdot \frac{1}{b} \cdot ab}$$ Distributive Property

$$= \frac{\left(\frac{1}{a} \cdot a\right)b - \left(\frac{1}{b}\ b\right)a}{ab - \left(\frac{1}{a} \cdot a\right)\left(\frac{1}{b} \cdot b\right)}$$ Commutative Property

$$= \frac{1 \cdot b - 1 \cdot a}{ab - 1 \cdot 1}$$ Multiplicative Inverse

$$= \frac{b - a}{ab - 1}$$ Multiplicative Identity

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